Number and volume raindrop size distributions in Taiwan

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Abstract:

Knowledge of rainfall characteristics is very important for the accurate estimation of rainfall kinetic energy and prediction of soil erosion. In this study, a reliable and efficient data collection and analysis system was developed to analyse the natural raindrop data collected in subtropical Taiwan. Both raindrop size distributions by number and volume were carefully analysed. The seasonal variations of the rainfall erosivity factor R, which is an index of the erosive potential of rainfall and a function of rainfall kinetic energy, was also discussed. An isoerodent map of Taiwan was also developed based on the rainfall data recorded by 158 automated rainfall-measuring stations within 26 years. Copyright © 2007 John Wiley & Sons, Ltd.

KEY WORDS raindrop size distribution; rainfall kinetic energy; soil erosion

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INTRODUCTION

Raindrop size distribution is useful information for estimating rainfall kinetic energy and predicting soil erosion. Natural rainfall has a wide distribution and varies with both space and time. It is difficult to collect rainfall data of natural storms with short duration and high intensity. Ample samples of raindrop data have to be analysed to determine the variability of raindrop size distribution at a given location. It is, therefore, necessary to develop a reliable and efficient data collection and analysis system.

Many methods have been used to measure the natural raindrops, e.g. 'filter paper' or 'stain' method (Wiesner, 1895; Hall, 1970; Cerda, 1997), 'flour pellet' method (Bentley, 1904; Laws and Parsons, 1943; Carter et al., 1974), optical, video or electro-mechanical distrometer or piezoelectric force transducer (Joss and Waldvogel, 1967; Coutinho and Tomás, 1995; Jayawardena and Rezaur, 2000), and photographical, radar or camera method (Eigel and Moore, 1983; Mueller, 1966), among others. Each of these methods has its advantages and disadvantages. The flour pellet method is to allow the raindrop to fall into a layer of fine, compacted flour and exposed to the rain, to produce the dough-pellets. These doughpellets are then extracted, labelled, and photographed for further analysis. The procedure of this method is very laborious. Also, multiple drops may simultaneously impact the same spot. As for the distrometer, it may have the following three disadvantages: (1) multiple drops of different sizes may simultaneously impact the transducer diaphragm; (2) unable to detect smaller drops due to the sensitivity problem; and (3) relatively expensive. With regard to the radar (or drop camera) method, liquid drops have been photographed successfully in the laboratory for many years (Worthington, 1908; Edgerton *et al.*, 1939; Best, 1947; Jones *et al.*, 1953). However, prior to the development of the Illinois raindrop camera (Mueller, 1966) no successful large field drop size camera had been reported. The data sets collected by Mueller (1966) at many locations in the US and at Bogor, Indonesia have been used by several researchers for erosion research (e.g. Kinnell, 1981; McIsaac, 1990). The drop camera as used by Mueller (1966) is capable of collecting ample samples within 1 m³ of air space in about 10 s. However, it is expensive and may also have raindrop overlapping problems.

Although the filter paper is one of the oldest methods, it is very accurate and stable. With proper modifications both on the raindrop-collecting box and the data analysing software, as will be described later, the proposed method will prove to be a reliable and efficient measuring system. In addition, it is cheap and easy to operate.

Lenard (1904) conducted one of the earliest experiments related to the raindrop size distributions. Marshall and Palmer (1948) proposed a famous exponential model (M-P distribution) to describe the raindrop size distribution. However, Mueller (1966) pointed out that the data analysed by Marshall and Palmer (1948) failed to include the smaller raindrops (less than 1 mm). Coutinho and Tomás (1995) also used the exponential distribution, instead of the gamma distribution suggested by Ulbrich (1983), to analyse their data collected with the RD-69 distrometer in southern Portugal. Since the maximum natural raindrop has an upper limit (unstable and breakdown problems), Bezdek and Solomon (1983) recommended the upper-limit log-normal distribution to describe the raindrop size distribution. Quimpo and Brohi (1986) reanalysed Mueller's (1966) raindrop size data and found

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that the log-normal distribution provided a better representation than all the other models examined (upper-limit log-normal, normal and exponential distributions).

Based on the raindrop size distribution data collected by Laws and Parsons (1943) at Washington DC, Wischmeir and Smith (1958) developed a semi-logarithmic relationship between rainfall kinetic energy and rainfall intensity. The relationship has been used for decades all over the world within the framework of the Universal Soil Loss Equation (USLE; Wischmeir and Smith, 1978). However, Kinnell (1981) suggested that one uses the following relationship:

$$E = e_{\max}[1 - a\exp(-bI)] \tag{1}$$

where *E* is the rainfall kinetic energy, e_{max} is the maximum kinetic energy, *I* is rainfall intensity, and *a* and *b* are constants. Van Dijk *et al.* (2002) performed a critical literature review on the rainfall intensity-kinetic energy relationships, including the exponential form (Equation (1)), semi-logarithmic relationship and a power law. It was found that the 'general equation' (Equation (1)) produced energy estimates that were within 10% of predictions by a range of parameterizations of the exponential model fitted to specific data-sets. However, in regions experiencing strong oceanic influence, at high elevations or at semi-arid to sub-humid locations, the equation may overpredict or underpredict the true rainfall energy.

Both the USLE and the revised USLE (RUSLE; Renard *et al.*, 1997) are widely used soil loss prediction models. The annual soil loss is expressed as a function of the rainfall erosivity factor, which in turn is a function of rainfall kinetic energy in these two models.

The main objective of this study is to analyse the raindrop size distribution based on the data collected in subtropical Taiwan with a reliable and efficient measuring system. The secondary objective is to analyse the seasonal variation of monthly rainfall erosivity and to develop an isoerodent map in Taiwan.

METHOD AND THEORY

Site description

Located in the subtropical area, Taiwan has ample rain, and its average annual rainfall is around 2500 mm. About 30% of the total area of the island is covered with mountains over the elevation of 1000 m, of which the geology is mostly composed of sedimentary and metamorphic rocks. Due to the frail geology, soil erosion is common during the rainy season.

Table I shows the geographic features of the four test regions from which samples of natural raindrops are analysed in this study. These test regions are respectively sited at Taichung (flat land) and Lien-Hua-Chi (mountainous area) in central Taiwan where the south-west monsoon influences the raindrops during summer, Keelung (flat land) and Yang-Ming Mountain (mountainous area) in the north where the north-east monsoon influences the raindrops during winter. Figure 1 illustrates the locations of the test regions, the average annual isohyets (unit: mm; 1975–2000), and three frequently occurring routes of typhoons near Taiwan (about 73% chance; Wang and Yi, 1990).

Experimental setup and procedure

In this study, natural raindrops are collected by means of dyed filter papers. The first step is to mix 1000 c.c. clear water with 1 g of chemical dye Rhodamine B (Basic Violet 10). The solution is then used to dye the pure white UK Whatman number 1 filter paper pink.

Figure 2 presents the raindrop-collecting boxes devised on the basis of camera shutter theory and a portable rain gauge. During the rainfall, three samples of natural raindrops are collected at the same time in order to increase the quantity of raindrop samples in a single case, and to improve the accuracy of raindrop analysis.

More specifically, the procedure for collecting raindrop samples in an open area is as follows: (1) put three dyed filter papers on the lower portion of the raindropcollecting boxes, and cover them with the upper portion (match A with A'); (2) start measuring the rainfall intensity with the portable rain gauge and a stopwatch; (3) place the raindrop-collecting boxes beside the rain gauge, and pull the upper portion of the boxes rapidly leftward (move opening A from A', A" to A"" in about 1 s); (4) record the reading of the rain gauge and the time on the stopwatch; (5) open the raindrop-collecting boxes in a dry area, remove the dyed filter papers for air-drying.

Figure 3 shows the relationship between the waterdrop size and the stained area. In our laboratory, the waterdrops are produced through a series of syringe needle heads of different diameters, and are regarded as spheres. Then they are allowed to fall freely from an airless tube of 9 m high, and drop on the dyed filter paper below the tube. By regression analysis, the relation of the two factors is:

$$D = 1.9576A^{0.394} \tag{2}$$

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Test region	Elevation (m)	East longitude	North latitude	Average annual rainfall (mm)
Yang-Ming Mountain (Chinese Culture University, Taipei)	607.1	121°32′10·58″	25°09′53.90″	4416.0
Keelung (National Taiwan Ocean University)	26.7	121°43′55.66″	25°08'05.18"	3782.7
Taichung (National Chung Hsing University)	34.0	120°40′33·31″	24°08′50.98″	1645.0
Lien-Hua-Chi (Taiwan Forestry Research Institute)	666.0	120°53′24·40″	23°55′28·30″	1920-3



Figure 1. Geographic locations of the test regions, the average annual isohyets in Taiwan (unit: mm; 1975–2000), and three frequently occurring routes of typhoons near Taiwan

in which *D* is the waterdrop diameter (in mm) and *A* is the stained area (in cm²). Equation (2) can be used to transform the stained area of natural raindrops collected through the dyed filter papers into raindrop diameter. Equation (2) was calibrated for waterdrop up to 6.3 mm in diameter (Maa, 1995). In fact, because of the high linearity (coefficient of determination $r^2 = 0.998$), the equation can be applied to most of the real situations.

Efficient raindrop image acquisition system

Using the 'efficient raindrop image acquisition system' developed by Borland C++ language for the Windows platform, the stains of natural raindrop size collected through the dyed filter papers can be calculated accurately.

Figure 4 illustrates the operating software in the windows. Based on the TWAIN_32 interface in the operation system, the software can be run with a colour scanner, and thus scans the stains on the dyed filter papers. The figure shows the software setup, including: (1) interface options: to set the scanning accuracy, graphics display method, and automated sequencing; (2) search options: to adjust stain colour range, scanning resolution, recognition method, maximum ratio of inner to outer circular area, minimum area, and minimum pixel points; (3) curvature comparison: to set raindrop radius range, maximum curvature ratio, and error elimination number. For the example in Figure 4, a 'scanner resolution' of 200 DPI is chosen, which corresponds to a minimum drop size of 0.13 mm (25.4 mm/200), and is very close to the critical minimum natural raindrop size of 0.1 mm (Chow et al., 1998). The 'curvature comparison' step is performed to eliminate the unreasonable raindrops which may be caused by the overlapping of the raindrops or incomplete



Figure 2. Raindrop-collecting boxes and portable rainfall gauge



Figure 3. Relationship between waterdrop diameter and the stained area on the dyed filter paper

raindrop stains near the periphery of the colour filter paper.

Rainfall erosivity

In the USLE (Wischmeir and Smith, 1978) or revised USLE (RUSLE; Renard, 1997), the rainfall erosivity factor (R-factor, an index of the rainfall erosive potential) is defined as:

$$R = \frac{EI_{30}}{100}$$
(3)

where I_{30} is the maximum 30-min rainfall intensity (which indicates the prolonged-peak rates of detachment and runoff). The value of the annual rainfall erosivity *R* can be obtained by summing up the EI_{30} values for all the rainstorms over a year. Similarly, the monthly *R* value can be obtained by summing up the EI_{30} values for all the rainstorms over a month. Since *E* is in hundreds of foot-tonf per acre and I_{30} is in inches per hour (in h⁻¹), based on Equation (3) the unit of annual *R* is (100 ft tonf in $ac^{-1} h^{-1} year^{-1}$).

Probability distributions for raindrop sizes

In this study, the distribution of natural raindrop size is discussed in two parts. The first one, whose calculation was based on the traditional statistical analysis of the numbers for different drop size classes, is called 'drop size distribution by number' in this study. The second, which is different from the former in the calculation of the statistics (mean, standard deviation, etc.) as the drop diameter data must be weighted by the drop volume, is called 'drop size distribution by volume' (Maa, 1995).

Assuming in a rainfall event the raindrop sizes are $x_1, x_2, x_3, \ldots x_n$, with regard to drop size distribution by number and that by volume, the mean (μ) and the standard deviation (σ) can be estimated respectively as:

(1) Drop size distribution by number:

$$\hat{\mu} = \overline{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \tag{4a}$$

$$\hat{\sigma} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \overline{x})^2}$$
(4b)

(2) Drop size distribution by volume:

$$\hat{\mathbf{u}} = \sum_{i=1}^{n} \left(x_i \times \frac{x_i^3}{\sum x_j^3} \right) \tag{5a}$$

$$\hat{\sigma} = \sqrt{\frac{n}{n-1} \sum_{i=1}^{n} (x_i - \overline{x})^2 \times \frac{x_i^3}{\sum x_j^3}} \qquad (5b)$$

In this study, normal distribution, log-normal distribution, gamma distribution, and beta distribution are examined for fitting the raindrop size distributions. The Kolmogorov–Smirnov test is then applied to compare their goodness-of-fit.

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Figure 4. Efficient raindrop image acquisition system

If the probability density function of 'drop size distribution by number' is $f_N(.)$ while that of 'drop size distribution by volume' is $f_V(.)$, their relationship is:

$$f_V(x) = \frac{x^3 f_N(x)}{\int_0^\infty x^3 f_N(x) dt}$$
(6)

According to Equation (6), when $f_N(x)$ follows four of the above mentioned distributions, $f_N(x)$ and $f_V(x)$ can be expressed as:

(1) Normal distribution with parameters μ and σ^2 :

$$f_N(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$
(7)

$$f_V(x) = (\text{constant})|x|^3 e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$
 (8)

(2) Log-normal distribution with parameters μ and σ^2 :

$$f_N(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{[\ln(x) - \mu]^2}{2\sigma^2}}, x > 0$$
(9)

$$f_V(x) = (\text{constant}) \frac{1}{x} e^{-\frac{1}{2\sigma^2} [\ln(x) - (\mu + 3\sigma^2)]^2}, x > 0$$
 (10)

in which f_V(x) is a log-normal probability density function while μ + 3σ² and σ² are its parameters.
(3) Gamma distribution with parameters η and λ:

$$f_N(x) = \frac{\lambda^{\eta}}{\Gamma(\eta)} x^{\eta-1} e^{-\lambda x}, x > 0$$
(11)

$$f_V(x) = (\text{constant})x^{(\eta+3)-1}e^{-\lambda x}, x > 0$$
 (12)

in which $f_V(x)$ is a gamma probability density function with $\eta + 3$ and λ as its parameters, and Γ is the gamma function.

(4) Beta distribution with parameters α and β in an interval (a, b):

$$f_N(x) = \frac{1}{B(\alpha, \beta)} \frac{(x-a)^{\alpha-1}(b-x)^{\beta-1}}{(b-a)^{\alpha+\beta-1}},$$

$$a < x < b$$
(13)

$$f_V(x) = (\text{constant})x^3(x-a)^{\alpha-1}(b-x)^{\beta-1},$$

$$a < x < b$$
(14)

in which $B(\alpha, \beta) = \Gamma(\alpha)\Gamma(\beta)/\Gamma(\alpha + \beta)$. In Equations (8), (10), (12), and (14), the constants make the integral value of $f_V(x)$ equal to 1.

Both the Kolmogorov–Smirnov (K-S) test and χ^2 test are frequently used goodness-of-fit tests, i.e. one-sample tests concerning the difference between an observed cumulative distribution of sample values and a specified distribution function. The K-S test is generally more efficient than the χ^2 test for goodness-of-fit, and is therefore adopted in this study. In addition, the parameter values for various distributions are estimated by the method of moment in this research.

RESULTS

Rainfall intensity-raindrop size relations

To take the rainfall event in Taichung test region for example, the relationship between the median volume drop diameter (D_{50}) and the rainfall intensity is given in Figure 5. Figure 6 shows the regression curves of the representative raindrop sizes ($D_5, D_{25}, D_{50}, D_{75}, \text{ and } D_{95}$) and the rainfall intensity, where D_n is the size of raindrop for which n% of sample by volume is finer. Since D_5 and D_{95} are very close to D_{\min} and D_{\max} , respectively, the results of Figure 6 reveal that the range of raindrop size distribution increases with an increase of the rainfall intensity. Listed in Table II are such items collected in each test region as rainfall intensity ranges and median, raindrop size range, the relation of median raindrop diameter and rainfall intensity, and the determination coefficient in the regression analysis. In the meantime, the relation for a significance level of 5% is also examined, and the results of the relations are good.

In Figure 7, all the relations of the median raindrop diameter and the rainfall intensity were graphed for different test regions, and compared with that of Laws and Parsons (1943). It can be inferred that the median raindrop diameter grows with the increase of the rainfall



Figure 5. Relationship between median volume drop diameter and rainfall intensity in Taichung test region

intensity. The tendencies of the two towards increase remain coherent.

Raindrop size probability distributions—by number and volume

Figure 8 shows the histograms of the raindrop size distributions both by number and by volume, and all the



Figure 7. Comparison of relationships between median raindrop diameter and rainfall intensity for different test regions



Figure 6. Relational curves of representative raindrop sizes and rainfall intensity in Taichung test region

Test region	Sample number	$I(\text{mm h}^{-1})$	D (<i>D</i> (mm)		$D_{50}{}^{a}$ (mm)		$D_{50} = aI^{\rm b}$		Significance ^c
			Min.	Max.	Min.	Max.	а	b		
Yang-Ming Mountain	113	0.73-67.1	0.67	4.75	0.70	3.50	0.978	0.249	0.52	*
Keelung	32	0.78 - 42	0.46	4.57	1.11	3.07	1.201	0.175	0.49	*
Taichung	128	0.71 - 150	0.66	5.24	1.13	3.87	1.211	0.176	0.52	*
Lien-Hua-Chi	67	0.89-111.7	0.80	6.02	1.00	4.50	1.436	0.133	0.34	*

Table II. Results of raindrop size analysis for the test regions

^a Laws and Parsons (1943): $D_{50} = 2.230I^{0.182}$

^b The unit of D_{50} is in mm, and that of *I* is in mm h^{-1} .

^c The asterisk means that for the significance level of 0.05, the relation is significant.





Figure 8. Histograms of the raindrop size distribution in Taichung test region and all the corresponding fitted probability density functions: raindrop size distributions by (a) number, (b) volume

corresponding fitted probability density functions for a typical rainfall event in Taichung test region. According to Figure 8, due to the large number of small raindrops, the raindrop size distribution by number tends to be leftskewed. However, after the data are weighted by the drop volume, the drop size distribution by volume tends to be nearly symmetric.

In this study, the most appropriate probability density functions are further discussed on the basis of natural raindrop size distributions both by number and by volume. By means of K-S test and setting the significance level of 5%, four theoretical probability distributions—normal distribution, log-normal distribution, gamma distribution, and beta distribution are examined. Tables III and IV show, respectively, the pass percentages of drop size distributions by number and by volume.

Table III indicates that the beta distribution best describes the natural raindrop size distribution by number in the four test regions because it gives the highest pass percentage for all four. Figure 9 shows the relationships between various parameters and rainfall intensity for the beta distribution in Taichung test region, which has widest range for the rainfall intensity (up to about 150 mm h^{-1}) among the four test regions. From

Table III. Results of the goodness-of-fit tests on the theoretical
probability models with regard to natural raindrop size distribu-
tion by number (K-S test, significance level of 0.05)

Test region	Sample number	Probability model	Pass percentage of drop size distribution by number		
Yang-Ming Mountain	113	Beta Log-normal Gamma Normal	82·3% 74·3% 71·7% 55·8%		
Keelung	32	Beta Log-normal Gamma Normal	84·4% 62·5% 59·4% 46·9%		
Taichung	128	Beta Log-normal Gamma Normal	90.6% 79.7% 76.6% 49.2%		
Lien-Hua- Chi	67	Beta Log-normal Gamma Normal	73.1% 52.2% 46.3% 32.8%		

Table IV. Results of the goodness-of-fit tests on the theoretical probability models with regard to natural raindrop size distribution by volume (K-S test, significance level of 0.05)

Test region	Sample number	Probability model	Pass percentage of drop size distribution by volume		
Yang-Ming Mountain	113	Beta Log-normal Gamma Normal	77.0% 71.7% 75.2% 77.0%		
Keelung	32	Beta Log-normal Gamma Normal	65·6% 59·4% 78·1% 71·9%		
Taichung	128	Beta Log-normal Gamma Normal	78·1% 75·8% 84·4% 82·0%		
Lien-Hua- Chi	67	Beta Log-normal Gamma Normal	59·7% 47·8% 65·7% 73·1%		

Figures 9(e) and 9(f) one can see that as rainfall intensity increases, the D_{\min} value only increase slightly, but the D_{\max} value increases significantly up to about 90 mm h⁻¹. As a result, Figures 9(a) and 9(b) indicate that both the sample mean and sample standard deviation increase with an increase of the rainfall intensity (*I*). However, from figures 9(c) and 9(d) both α and β values, which were estimated by the method of moment, remain almost unchanged as the rainfall intensity increases.

The pass percentage of log-normal distribution is the second best. These results correspond to those of Quimpo and Brohi's (1986) research. Of the four models for raindrop size frequency distributions (exponential, normal,



Figure 9. Relationships between various parameters and rainfall intensity for beta distribution and Taichung test region

log-normal, and upper limit log-normal) considered, they found that the log-normal model appears to better fit most of their data.

With regard to the pass percentage of natural raindrop size distribution by volume in Table IV, as the data weighted by drop volume tend to be nearly symmetric, the pass percentage of normal probability distribution, in general, is the highest.

Through the K-S test on the natural raindrop size distribution by number or volume, it can be found that the raindrop size distribution during lighter rainfall tends to be more continuous and smooth. Therefore, it is easier to pass the test during lighter rainfall. However, as the rainfall intensity increases, the raindrop distribution tends to be less continuous (e.g. bimodal). As a consequence, the pass percentage tends to decline gradually.

APPLICATIONS

Due to the greenhouse effect in recent years, the global climate is undergoing drastic changes, which are causing direct impacts on our living environment. The local features of rainfall in Taiwan have also changed. Figure 10 shows the average annual rainfall in Taiwan within 26 years (1975–2000) and the 5-year moving average. It reveals that the rainfall tends to increase gradually in



Figure 10. Variations of the average annual rainfall between 1975 and 2000, and the 5-year moving average curve

recent years (last six 5-year moving average points, representing 1993–2000).

Based on the methodology developed in this study, the rainfall intensity-kinetic energy relationships for different test sites were established. The rainfall erosivity (R



Figure 11. Comparison of the monthly variations of the rainfall erosion index for different test regions

value) for each storm, which is defined as the product of a storm's kinetic energy and the maximum 30-min intensity times 10^{-2} (see Equation (3)), was then accurately calculated.

Monthly variations of erosivity

Figure 11 shows a comparison of the monthly variations of rainfall erosivity for the test regions, indicating a drastic seasonal variation of the rainfall erosivity. The monthly R values were calculated from Equation (3), in which I_{30} values were estimated from the original rainfall recording charts provided by the Central Weather Bureau of Taiwan. For Taichung and Lien-Hua-Chi, which are located at the central area of Taiwan (west of Central Mountains), the R values are relatively high during the summer (June-August) due to the effect of typhoons. However, for Yang-Ming Mountain and Keelung, which are located near the northern area of Taiwan, the R values are relatively high during summer and autumn (June-October) due to the combined effects of typhoons and monsoons.

Regional variations of erosivity

In addition, with consideration of both the topographic and geographic factors, in this study Taiwan is divided into 10 climatic regions (Lu *et al.*, 2005). The relationships of rainfall erosivity and annual precipitation (R-P relations) for 10 representative regions were also established. The rainfall data collected by the Central Weather Bureau and the Water Resources Agency at 158 automated rainfall-measuring stations during 1975–2000 were also analysed. The *R* values for any other station was then calculated based on the appropriate R-P relation, depending on which climatic

region the station belongs to. By calculating the average annual rainfall erosion index (R-factor), the isoerodent map of Taiwan was developed, as shown in Figure 12. The plotted lines on the map connect points of equal rainfall erosivity. Erosion index values for locations between the lines are obtained by linear interpolation. The map in conjunction with the USLE or RUSLE can be used to estimate the average annual soil loss in Taiwan. Figure 12 clearly reveals the regional variation of R factor. Three frequently occurring routes of typhoons near Taiwan are also plotted in Figure 1. By comparing Figures 1 and 12, it can be seen that the routes of typhoons were affected by the high mountains, and the R values were clearly affected by the typhoons, which usually carry significant amount of rainfall. Also, many local maximums occur in the mountainous regions.

Furthermore, it is worth mentioning that the averageannual values of the erosion index in most areas of subtropical Taiwan are greater than 500 (100 ft tonf in $ac^{-1} h^{-1} year^{-1}$), while it is less then 550 (100 ft tonf in $ac^{-1} h^{-1} year^{-1}$) in most areas of the US (Wischmeier and Simth, 1978). Hence, serious soil erosion hazards may occur when the vegetal cover is removed due to either the natural causes or human activities, e.g. earthquake or highway construction.

SUMMARY AND CONCLUSIONS

- 1. In this study, an efficient natural raindrop size image acquisition system was developed. It can be used with a portable raindrop-collecting box and a rain gauge to collect and analyse natural raindrop size at reasonably low cost.
- 2. The probability distributions of natural raindrop size in the four regions of subtropical Taiwan were analysed. With regard to the drop size distribution by number, the beta distribution is the most descriptive. As to the drop size distribution by volume, since the data weighted by the drop volume tend to be nearly symmetric, the pass percentage of the normal distribution remains the best in general.
- 3. Based on the methodology developed in this study, the rainfall intensity-kinetic energy relationships for different test sites were established. A comparison of the monthly variations of rainfall erosivity for different test sites indicates a drastic seasonal variation of the R values, which is mainly caused by the combined effects of typhoons and monsoons.
- 4. According to the rainfall data recorded by 158 automated rainfall-measuring stations within 26 years (1975–2000), an isoerodent map of Taiwan was developed. The map in conjunction with the USLE or RUSLE can be used to estimate the average annual soil loss in Taiwan. The map clearly reveals the drastic regional variation of the rainfall erosivity in the subtropical Taiwan.



Figure 12. Isoerodent map of Taiwan (unit: 100 ft tonf in ac⁻¹ h⁻¹ year⁻¹; 1975-2000)

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APPENDIX

Proofs of Equations (5) and (6)

function (c.d.f.) of the size distribution by volume is

$$F_V(d) = \frac{\text{Volume for sizes} \le d}{\text{Total volume}}$$

 $=\frac{\displaystyle\int_{0}^{d}\frac{\pi}{6}x^{3}f_{N}(x)\mathrm{d}x}{\displaystyle\int_{0}^{\infty}\frac{\pi}{6}x^{3}f_{N}(x)\mathrm{d}x}$

(Assuming spherical shape)

$$= \frac{\int_0^d x^3 f_N(x) \mathrm{d}x}{\int_0^\infty x^3 f_N(x) \mathrm{d}x}$$

Proof of Equation (6). The cumulative distribution Thus the probability density function (p.d.f.) of the size distribution by volume is

$$f_V(x) = F'_V(x)$$

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$$=\frac{x^3 f_N(x)}{\int_0^\infty x^3 f_N(x) \mathrm{d}x} \tag{6}$$

Proof of Equation (5).

For
$$f_V, E(x) = \frac{\int_0^\infty x^4 f_N(x) dx}{\int_0^\infty x^3 f_N(x) dx}$$

Therefore, the mean E(x) can be estimated by

$$\hat{\mu} = \frac{\frac{1}{n} \sum x_i^4}{\frac{1}{n} \sum x_i^3} = \sum_{i=1}^n x_i \cdot \frac{x_i^3}{\sum_{j=1}^n x_j^3}$$
(5a)

Similarly, the standard deviation can be estimated by

$$\hat{\sigma} = \frac{n}{n-1} \sum_{i=1}^{n} (x_i - \overline{x})^2 \cdot \frac{x_i^3}{\sum_{j=1}^{n} x_j^3}$$
(5b)

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